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Analysis of the Dynamic Behaviour of a Foundation Subjected to Machine Induced Vibrations

Dr.Venkat Praveen Gannavaram[1], M. Uday Bhaskar[2], Bukke Lavanya[3]

[1]*Professor, Department of Civil Engineering, Sreenidhi Institute of Science and Technology(Autonomous), Ghatkesar, Hyderabad, Telangana-501301*

[2]*Asst.Professor, Department of Civil Engineering, MallaReddy Engineering College (Autonomous) Maisammaguda, Dhulapally, Secunderabad, Telangana-500100.*

[3]*Asst.Professor, Department of Civil Engineering, Sreenidhi Institute of Science and Technology(Autonomous), Ghatkesar, Hyderabad, Telangana-501301*

Abstract: Machine foundations require a special consideration because they transmit dynamic loads to soil in addition to static loads due to weight of foundation, machine and accessories. The amplitude of vibration of a machine at its operating frequency is the most important parameter to be determined in designing a machine foundation, in addition to the natural frequency of a machine foundation soil system. In this paper, three different case studies are considered, each with different constraints on the operating frequency, dimension of the foundation and dynamic properties of the supporting soil. This paper explores the advantages of using Finite Element analysis in addition to classical tables and analytical solutions, to analysis and design foundations subjected to machine induced vibrations.

Keywords: *Dynamic loads, Static loads, Finite Element Analysis, Foundation.*

1. Introduction

Foundations subjected to dynamic loads oscillate in a way that depends on the nature and deformability of the supporting ground, the geometry and the inertia of the foundation and the superstructure, and the nature of the dynamic excitation. The basic goal in the design of a machine foundation is not only to limit its amplitudes, which will endanger the satisfactory operation of the machine but also to prevent it from disturbing the people working in the immediate vicinity. Thus, a key ingredient to a successful machine foundation design is the careful engineering analysis of the foundation response to the dynamic loads from the anticipated operation of the machine. Furthermore, when excessive motions of an existing foundation obstruct the operation of the supported machinery, analysis is necessary in order to understand the causes of the problem and hence to guide appropriate remedial action. The theory of analyzing the forced vibrations of shallow and deep foundations hasadvanced remarkably in the last couple of decades and has currently reached a mature state of development. A number of formulations and computer programs have been developed to determine in a rational way the dynamic response in each specific case. Numerous studies have been published exploring the nature of associated phenomena and shedding light on the role of several key parameters influencing theresponse.

Solutions are also presently available in the form of dimensionless graphs and simple mathematical expressions from which one can readily estimate the response of surface, embedded and pile foundations of various shapes and rigidities, supported by deep or shallow layered soil deposits. On the other hand, little if any progress has been made in reliably estimating dynamic machine loads and improving (through calibration with field data) the available performance criteria [1, 2, 3].

The design of a machine foundation is a trial-and- error process involving the following engineering tasks:

a. Estimation of the magnitude and characteristics of the dynamicloads.

b. Establish the soil profile and determine the appropriate shear modulus and damping for each soillayer.

c. Selection of a trial dimension based on experience and in cooperation with the client established performance criterion and also depends on the type of the machine the foundationsupports.

d. Estimating the dynamic response of the trial foundation using Finite Element Analysis / analytical solutions or using an idealized spring-dashpot model

e. Check if the estimated response amplitude of the foundation-soil system conforms to the performance criterion. Repeat steps a – d, until a satisfactory design isobtained.

f. The design process usually stops here. However, in case of important projects, it is important to measure the actual performance of the constructedfoundation.

In some cases, the client criteria may also constrain the dimension of the foundation by the type of machine to be mounted, in which case one might try to replace the top 1 – 3m of soil layer to alter the dynamic behaviour of the supporting soil to keep the natural frequency of the soil-foundation system away from the operating frequency of the machine. In this paper, three different case studies on design of machine foundations are discussed. In certain cases, innovative approaches might be required to overcome the limitations based on the client criteria and siteconditions.

Case Study I: Dynamic Analysis and Design of Foundation Supported on Piles

The Primary Air (PA) and Force Draft (FD) fan foundations are to be designed for a thermal power plant in South India. The PA and FD fans are to be founded at a rocky site. The PA/FD Fan foundations are 11.75m x 3.5m in size and are to be founded at a depth of 1.5m below ground level. The operating frequency of the PA and FD fans is between 16Hz to 25Hz. The frequency of the fan soil -foundation system should be separated from the operating frequency of the PA and FDfans.

Site Characterization

The top 3.0m of the site is characterised by Medium Rock with RQD increasing from 12.5% to 55%. A core recovery of 60 to 90% was recorded in this layer. It is followed by a 5m thick highly weatheredrocklayer,withacorerecoveryof<27%. This layer is followed by a layer of Fractured Rock with a core recovery of about 70% and a RQD of 27%.Cross-hole test was carried out to determine the variation in the shear wave velocity with depth. The results of the cross-hole test are presented in Table 1. The Shear wave velocity increasesfrom about 500 m/s at a depth of 3m to 1500 m/s at a depth of 6m. The shear wave velocity varies between 1100 m/s and 1500 m/s within a depth of 8 to 10 m

Undertuned foundation

In this case, it's impossible to modify the size of the foundation due to machine requirements, hence alternative solutions such as altering the ground conditions to achieve the required soil-foundation stiffness was considered. Initial design calculations revealed that a low tuned foundation is feasible to meet the dynamic criteria. It was found that the foundation resting on the composite medium consisting of top 4.5m and 5.0m well-compacted fill for PA and FD fan foundation on the existing weathered rock to be a possibleoption.

Table 1. Shear Wave Velocity and Maximum Shear Modulus at PA/FD Site

Type of Soil /	**Depth**	**Vs**	**Gmax**
Rock		**(m/s)**	**(MPa)**
Weathered Rock	3.0	500	525
Weathered Rock	4.5	882	1713
Fractured Rock	6.0	15000	5175
Fractured Rock	8.0	11111	2840
FracturedRock	10.0	15000	5175

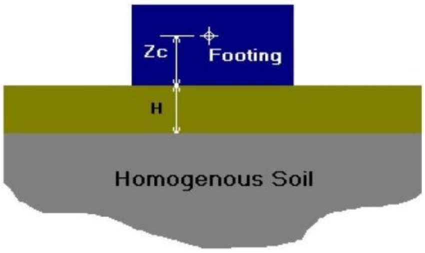

Fig. 1 Rigid footing resting on composite strata

The stiffness for the PA and FD Fan foundations was evaluated using advance DYNA 5.4 software developed by Geotechnical Research Centre of University of Western Ontario (Canada). The stiffness of the soil foundation system was evaluated considering a rigid surface footing resting on the composite soil strata consisting of a layer overlying a half space as shown in Fig.1.

To achieve the required stiffness, it was required to replace the existing fill / weathered rock with well graded gravel (GW) compacted to 95% relative density to achieve a shear wave velocity of about 300 m/s at a depth of 5.0m below the base of the PA and FD fan foundations.

The stiffness of the foundation-soil system is evaluated assuming the footing to be resting on the 4.5m thick well graded gravel layer with shear wave velocity of 300m/s above an elastic half- space of weathered rock with shear wave velocity of 700m/s. The total vertical and horizontal stiffness of the PA Fan foundation was found to be $6.29x10^{5}$kN/m and $1.66x10^{5}$kN/m respectively.

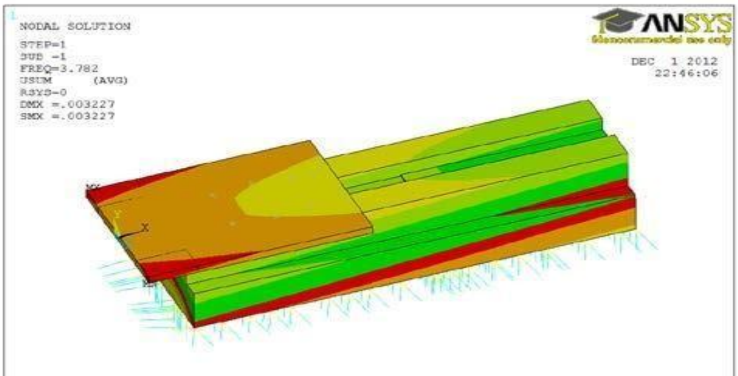

Fig. 2. Nodal displacement in the first mode – PA fan foundation on sand

Finite Element Analysis

A FE analysis was performed on the foundation with the spring constants obtained by assuming foundation resting on a composite layer above elastic half-space. The fan foundations were modelled using SOLID elements. The vertical and horizontal stiffness values were assigned to the base of the foundation using SURF154 elements. A modal analysis using Block Lanczos Technique was performed and the first 20 modes of vibrations were extracted. It was observed that the frequencies of the foundations are well separated from the operating frequency of the PA and FD fans. The displacement of the PA foundation resting on sand in its first modal frequencyis presented in Figure 2. The first modal frequency is 2.792Hz, which is away from the operating frequency of 16 to 25Hz

Rock Socketed Pile Foundation

Although PA/FD Fan resting on a dense fill was able to separate the frequency of the foundation from the operating frequency, it was considered impractical for the given site conditions. Hence, the foundation supported on rock socketed piles was considered to be a viable solution. To evaluate the vertical stiffness, the pile is assumed as an end bearing pile carrying a vertical load. The horizontal stiffness is obtained assuming the pile is free from the surrounding soil and fixed at the top and bottom [4]. The M25 grade concrete pile of 0.45m diameter and 11.5m in length is assumed to be socketed for a length of 1.5m below 10.0m GL (ground level). 15 Piles were used to support the PA/FD Fan, each supporting a load of 192 kN. The effective length of the pile is 10.0m. The vertical and horizontal stiffness of the pile is computed as1.8 x 106kN/m and 2.7 x104kN/m, respectively. The bearing capacity is calculated assuming the pile is socketed to weathered rock at 10m below GL with RQD of 0.1 [5]. Assuming the pile to be end bearing the allowable load carrying capacity was estimated as 450 kN. The lateral capacity of the pile subjected to 5mm lateral displacement is estimated as 3.6kN. The pile was found to be safe against buckling failure. Researchers observed that the site conditions, and in particular, the properties of the topsoil layers greatly govern the degree of non-linearity and the dynamic lateral stiffness of the soil-pile system [6]. To achieve the required stiffness, 800mm diameter bore hole is to be drilled up to the surface of medium rock (i.e. up to a depth of 10m from FGL or EGL) following which 450mmdiameter boring would be drilled for a length of 1.5m. Thus the rock-socketed pile would have a freestanding length of 10m.

A Finite Element analysis of the PA and FD fan foundations was performed assuming the piles to be linear springs with one end fixed and other end carrying a load. The fan foundations were modelled using SOLID elements and the 15 piles were modelled as springs using COMBIN14 elements. Both horizontal and verticalstiffness were accounted for using 2 horizontal and 1 vertical spring elements for each pile. The FE mesh with the springs is shown in Fig. 3.

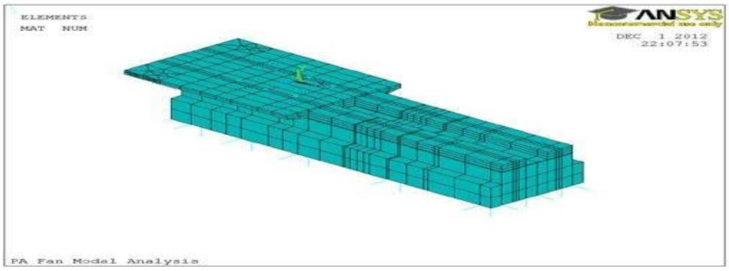

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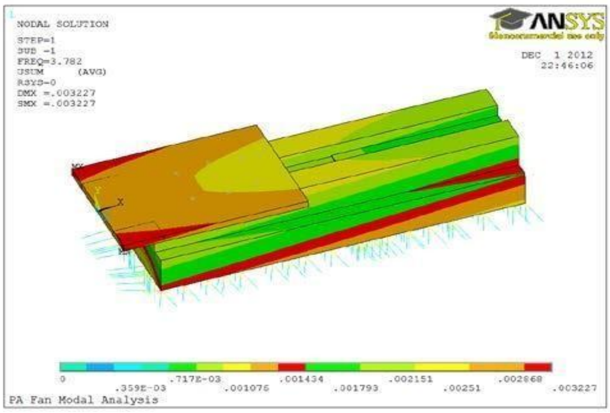
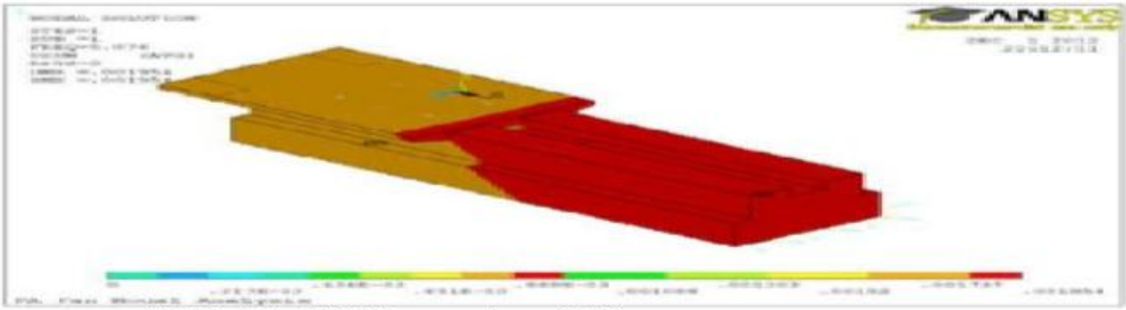
Fig. 3. Mesh of PA Fan foundation with Piles modelled as spring elements

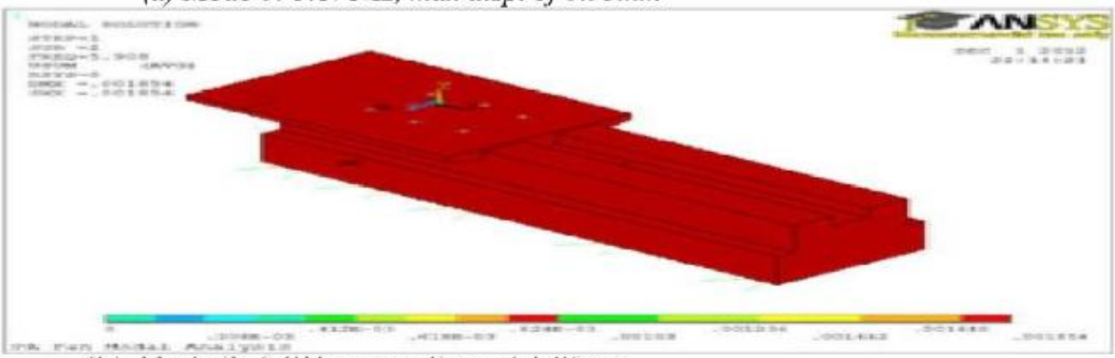
Modal analysis was performed to estimate the natural frequencies of the PA and FD fan foundation pile system. 20 modes were extracted using Block Lanczos approach. All the modes were found to be well separated from the operating frequencies of the PA and FD fans (16 to 25Hz).



The displacement profiles of the PA fan in the first four modes are presented in Fig. 4. It can be observed that the fan foundations are under-tuned and the natural frequencies are well separated from the operating frequencies of the fans. First mode corresponds to rocking displacement and the second mode corresponds to translational displacement. The lateral displacement observed in the second mode is well within the allowable displacement and the lateral capacity of pile. Thus a simple approach used to determine the stiffness of the pile

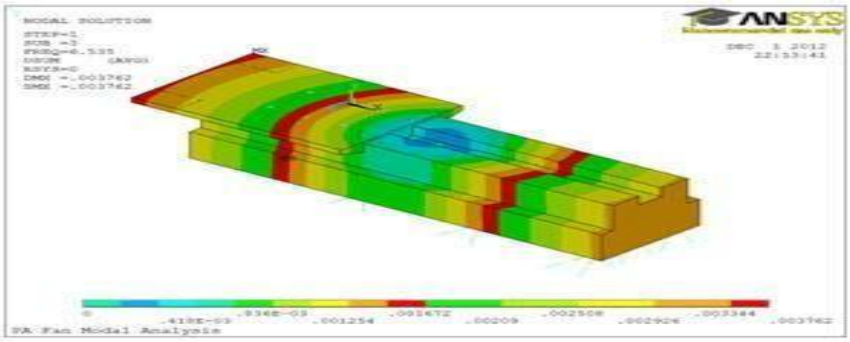


(a) Mode 1: 5.876Hz, max disp. of 1.95mm

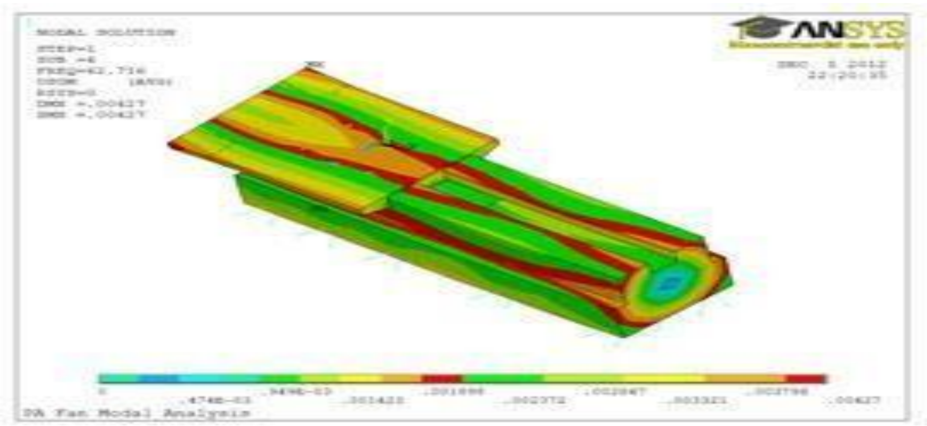


(b) Mode 2: 5.9Hz, max disp. of 1.85mm

is coupled with FE analysis to determine the natural frequency of the soil-pile-foundation system.



(c) Mode 3: 6.5Hz, max disp. of 3.76mm



(d) Mode 4: 62.72Hz, max disp. of 4.2mm

Fig. 4. First four modes of vibration for PAfanfoundation on piles

The first mode of vibration is rocking and has a modal frequency of 5.86Hz, which causes a maximum displacement of about 2mm. The second (translation) and the third mode are below the operating frequency of the machine and the fourth mode has a frequency of about 63Hz, which is well above the operating frequency of 16 – 25 Hz. The vertical amplitude at the top of the foundation is presented in Fig. 5.

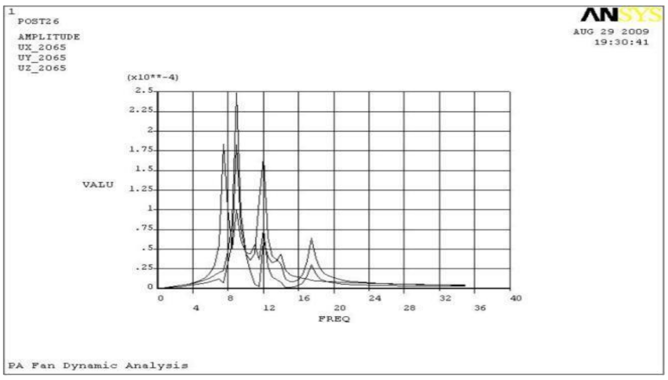


Fig. 5. Displacement amplitude for different modes of vibrations at the top of the foundation

Case study II: light cone penetration test (lcpt)

A foundation of 0.94m x 0.72m in dimension is to be constructed to support a dynometer-engine that operates at a frequency of 157 – 380 Hz. In this study, the dimension of the foundation was constrained due to lack of space. Hence, the options available to achieve the safe bearing capacity and to separate the natural frequency of the soil-foundation system from the operating frequency of the machine were: (a) to vary the height of the foundation to achieve the required frequency separation, increase in mass reduces the frequency in turn aids in designing the foundation as under-tuned, however it is important to consider the bearing capacity failure; (b) to modify the top few meters of the soil layer to achieve the required dynamic properties for the supportingsoil.

Site Characterization

No bore-log or soil profile data wasavailable; hence Light Cone Penetration Test was carried out at the site for a depth of 3m from 1m below the existing floor level to determine the variation of the soil profile below the proposed foundation. A 30 mm diameter cone connected to 25mm diameter sounding rods is driven into the soil by hammering. A 10 kg hammer is used to drive the cone and the blow counts for each 10 cm penetration are recorded. The cone is continuously driven to the required depth. The test procedure is very similar to dynamic cone penetration test described in IS 4968 (Part1) 1976. The variation of LCPT blow count 'n' with depth 1m below the existing floor level is presented in Figure6.

The LCPT profile reveals that the soil is very soft in the top 1m below the excavation level and is relatively dense below 1m. The relatively dense strata available at 2m below the existing floor level is found to extend to a deeper depth. A correlation between the SPT 'N' and LCPT 'n' as given below is used to obtain the corresponding SPT N values for determination of other geotechnicalparameters.

$$SPT\ 'N' = 0.411\ n \qquad (1)$$

The variation of SPT 'N' with depth is also presented in Fig. 6. It is evident from the figure that the SPT 'N' of <2 indicates presence of very soft clay deposit for a depth of 2m below Finished Floor level. The SPT 'N' is found to increaseswith depth. The soil samples collected during testing indicates that the soil is mainly composed ofclay.

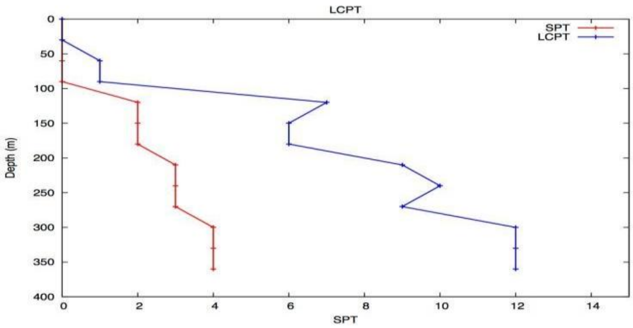


Fig. 6.Variation of LCPT blow count withdepth

The Atterberg limits tests were carried out on the collected samples in the laboratory. The atterbergs limits obtained are summarized below.

Liquid limit = 36.5%

Plastic limit =18.4%

Plasticity index =18.1

The soil is classified as Clay of low plasticity (CL) as per Indian standard.

Bearing Capacity

The LCPT test reveals that the top 2.5m soil layers to be very soft clay and hence the minimum depth of the foundation should be 2.5m. The bearing capacity evaluated based on the SPT 'N' is found to be a low value of 45kPa and is not suitable to support the foundation directly, hence it is recommended to excavate the existing soft clay soil for a depth of 2.5m and replace it with suitable soil. Based on the various design calculations, it is decided to provide an isolated footing resting on the fill material for the engine foundation to meet the static and dynamic criteria. Several options were considered taking into account the depth of soft clay soil and other foundation criteria. It is found that the foundation with 1m depth resting on a composite medium of 1.5m well-compacted fill for engine foundation resting on the existing soft clay to be a viable solution. It is ideal to replace the existing soil of 2.5m x 2.5m x 2.5m depth with well graded sand, however considering the space constrain on the side near dynometer the pit shall be restricted to 1.92 x 2.5 x 2.5m depth.The bearing capacity of the soil is evaluated considering the following details:

- FillMaterial =W ell graded Sand(SW)
- Thickness of the fill = 1.5m below GL
- SPT 'N' = 15
- Unit weight of fillmat erial $=18kN/m^3$

- Safe BearingCapacity = 155 kPa

The safe bearing capacity thus evaluated is found to meet the required static criterion to support the enginefoundation.

Stiffness Evaluation

The stiffness for the engine foundation is evaluated using DYNA 5.4 software. The stiffness of the soil foundation system was evaluated considering a rigid surface footing resting on the composite soil strata consisting of a layer overlying a half space as shown in Fig. 1. The following soil, foundation and machine parameters are used to evaluate the stiffness.

1. First Layer –well compacted wellgraded sand (SW)

a. Shear wave velocity = 200 m/s

b. Mass density =1800kg/m^3

c. Poissons ratio =0.3

d. Thickness of the layer = 1.5m (below base of thefoundation)

2. Second layer- existing softclay

a. Shear wave velocity = 115m/s.

b. Thickness = Halfspace

3. Foundation

a. Size = 0.94 m x 0.72m

b. Founding depth =1.0 m belowFGL.

4. Operating Frequency of Machine: 157-380 Hz.

The following soil stiffness constants are obtained from the analysis.

Total vertical spring constant = 1.4 x 10^4kN/m

Total horizontal spring constant = 1.2 x 10^4kN/m

Finite Element Analysis

The foundation is checked for overturning moment, sliding and bearing pressure. Theanalysis was done under the following load conditions as specified by the client. The forces considered in the analysis are presented in Table 2

Load Description	Value
Vertical vibrating load at the engine base level	42.2 kN
Horizontal vibrating force at the engine base	22.7 kN
Moment at base of engine	54.3 kNm
Dead weight ofengine (Max)	180 kg
Dead weight of baseplate	350kg

Table 2. Loads acting on the foundation

The static, modal and dynamic analysis of the foundation is performed using 3D finite element analysis software MECHANICAL APDL (ANSYS12). The footing is idealized as a linear elastic isotropic model (Fig. 7) with Young's modulus of 30x109 N/m2 and Poissons ratio of 0.3. The density of concrete is assumed as 25 kN/m3. The base of the foundation is modeled using 3D surface element SURF154 with elastic foundation stiffness. The vertical and horizontal stiffness of the foundation are arrived based on the equivalent spring constants for rigid foundation resting on composite system as arrived using DYNA.

The soil is modeled as linear elastic material withshear modulus of 72 MPa and Poissons ratio of 0.3. The shear modulus is calculated using the followingformula:

$$G = \rho \text{ x } V_s^2 \tag{2}$$

The footing is modeled using eight noded solid brick elements (SOLID 185) having three degrees of freedom at each node (translations in the nodal x, y, and z directions). The base of the footing is restrained in vertical directions. Free meshing was performed with fine level of smart sizing option available in ANSYS. The FE model used in the analysis is shown in Fig.7.

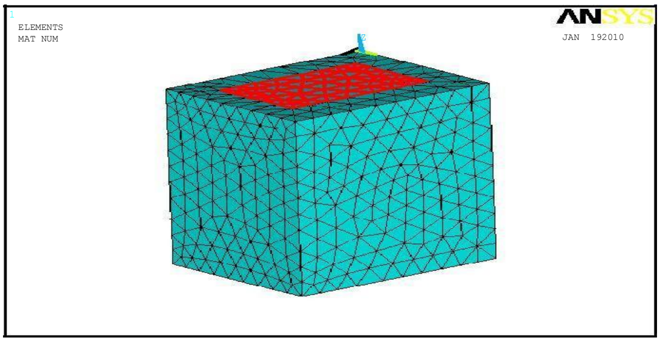


Fig. 7. Finite Element model of machine foundation

The static analysis was carried out to determine the maximum displacement due to static loading. The static analysis was performed by applying a pressure equivalent to the axial load acting on the top of the footing. The deformed shape of the foundation is presented in Fig. 8. The maximum displacement under static load is found to be 0.043 mm and is within permissible limits.

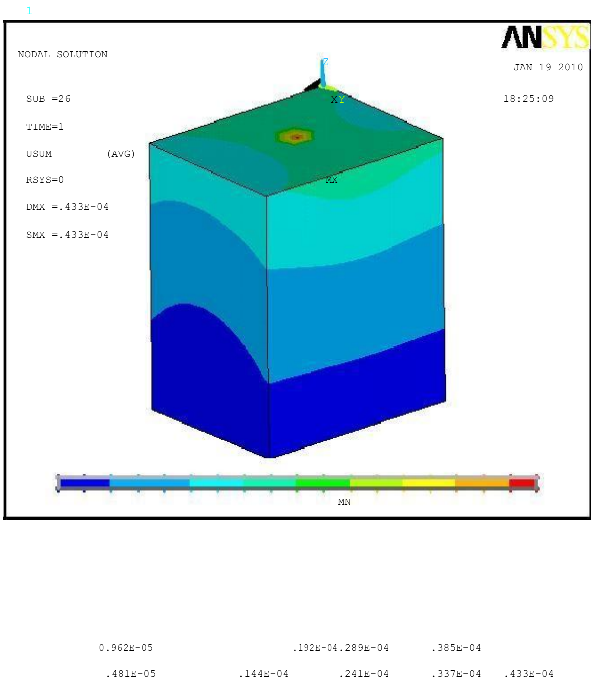


Fig. 8. Deformed foundation under static loading

The modal analysis of the foundation is carried out to determine the vibration characteristics (natural frequencies and mode shapes) of the foundation. Block Lanczos Mode extraction method was carried out and 20 modes were extracted. The modal frequencies of the system are obtained in the range of 132 - 683 Hz. The operating frequency of the engine is 150Hz to 380 Hz as specified by the client. A typical deformed mode shape corresponding to the first modal frequency is shown in Fig.9.

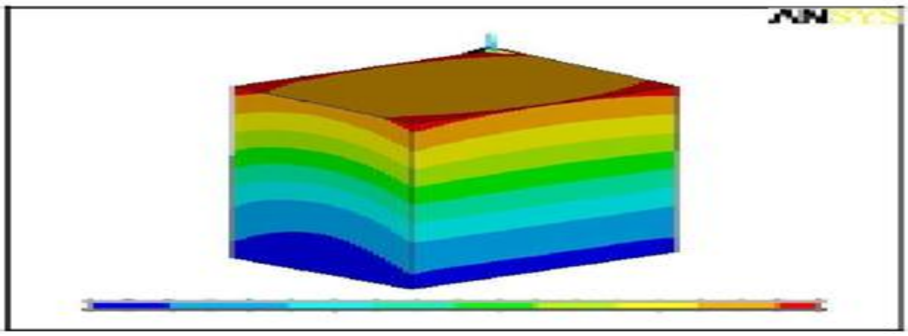


Fig. 9. Deformed mode shape corresponding to the first modal frequency

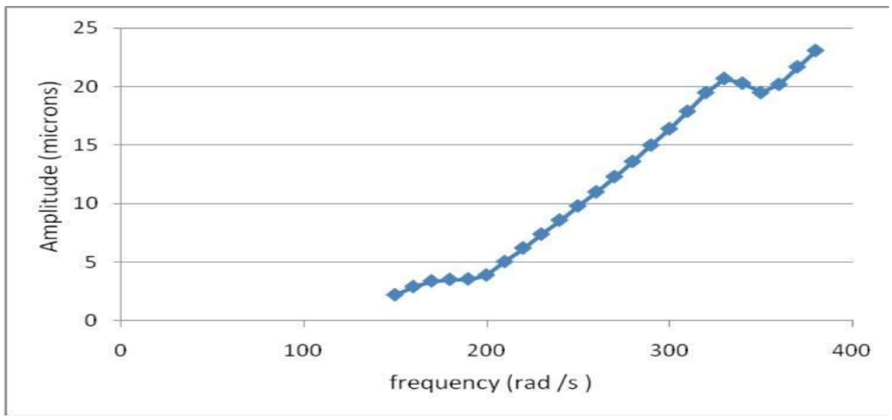


Fig. 10.Variation of maximum amplitude with frequency

The harmonic analysis of the foundation is carried out to determine the amplitude of vibration during the functioning of the engine. Harmonic load was applied as ramp with frequency range from 150 to 400 Hz. The variation of the maximum amplitude with frequency is presented in Fig. 10. The deformed shape corresponding to a frequency of 380 Hz is shown in Fig. 11. It is found that the maximum deformation of 0.023 mm occurs at a frequency of 380 Hz and is found to be within 1.0 mm of allowable deformation as specified by the client.

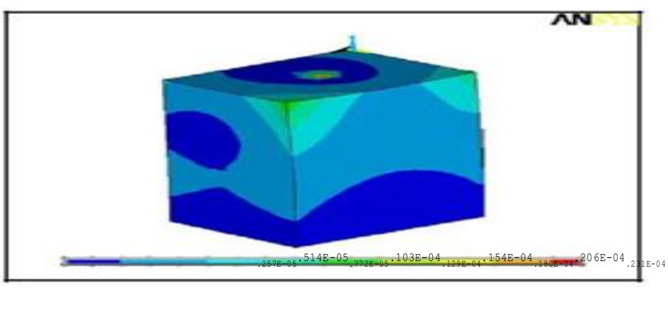


Fig. 11.Displacement vector sum at frequencyof 380 Hz.

Case Study III: Wind Turbine Foundation

In this case study, we explore the design process of a wind-turbine foundation for a given site condition.

The foundation for 1 MW wind turbine is to be designed for wind-farm at Thoothukudi District. Wind Turbine is a rotating machine that generates electricity by transforming the wind energy to electrical energy. Foundations for wind-turbines are subjected to resonance from the blades passing the columns in addition to the externalloads.

Site Characterization

The soil investigation report indicates a hard stratum at a depth of 1.6m with SPT "N" value of100. The safe bearing capacity of the soil at a depth of 1.6m from ground level is recommended as 250 kN/m2. The depth of the foundation is calculated as 3.0m.

Analysis

The foundation is checked for overturning moment, sliding and bearing pressure. The analysis was done under the following load conditions as specified by the Client. The following forceslike,(1) Normal Load (Nd), (2) moment (Md), (3) horizontal shear (Sd) and (4) torsion in tower centre axis (M) were considered in the analysis of the foundation of the wind turbine generator (Table 3).

Design

An octagonal raft of 14m diameter is designed to resist normal load from the tower mass and concrete pedestal, maximum bending moment, torque and shear due to the rotary movement in the turbine. The pedestal is designed for the axial load and maximum bending moment and checked against punching shear failure. The foundation is provided with 150 mm thick base sand pad for uniform distribution of stresses to the soil.

Finite Element Analysis

The static and dynamic analysis of tower foundation is performed using 3D finite element analysis software MECHANICAL APDL (ANSYS 12). The octagonal footing is idealized as a linear elastic isotropic model (Fig. 12) with Young's modulus of 2.6*105kN/m2 and Poissons ratio of0.3. The density of concrete is assumed as 25 kN/m3. The base of the foundation is modeled using 3D surface element SURF154 with elastic foundation stiffness of 6.8e+6 kN/m. The soil is modeled as linear elastic material with shear modulus of 170 MPa and Poissons ratio of 0.33. A non-solving 3D MESH FACET model was used to transfer the moments and torsion acting on the turbine to the foundation. The footing is modeled using eight noded solid brick elements (SOLID 185) having three degrees of freedom at each node (translations in the nodal x, y, and z directions). The base of the footing is restrained in all directions. Free meshing was performed with fine level of smart sizing option available in ANSYS. The FE model used in the analysis is shown in Fig. 12.

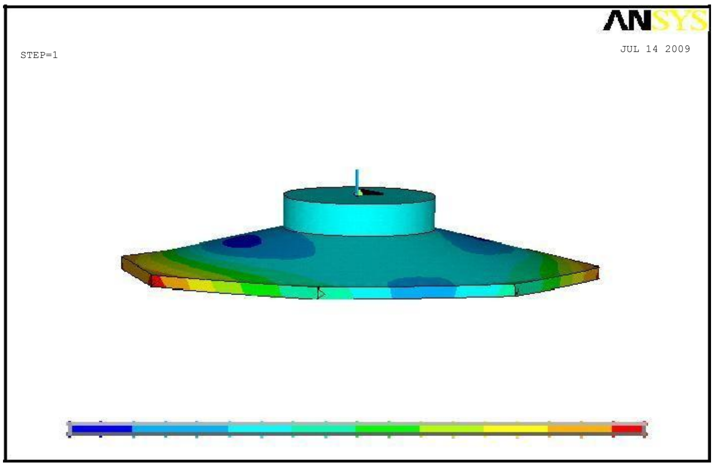


Fig. 12. FE mesh of the octagonal mesh with the column

The static analysis was performed by applying a pressure equivalent to the axial load acting on the top of the footing. The moments, torsion and shear are applied using MESH FACET (dummy elements). The inertia effect due to gravity is also considered. The deformed shape of the foundation is shown in Fig. 13.

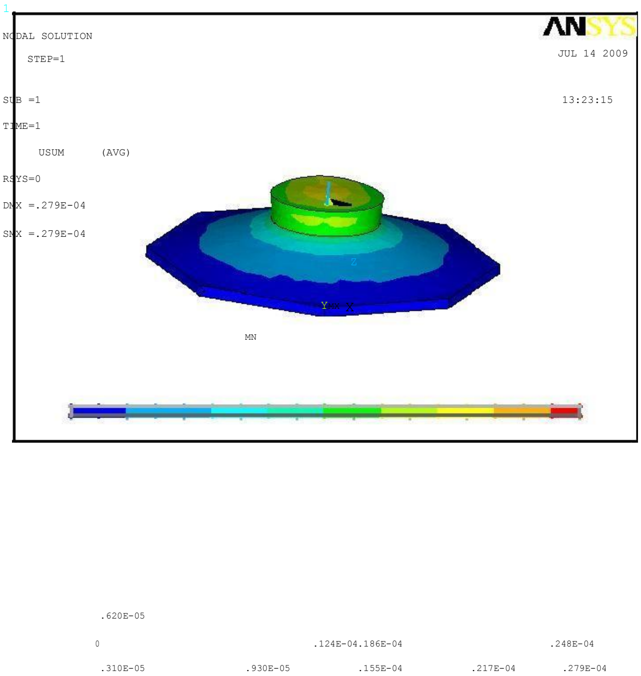


Fig. 13. Deformed shape of the foundation

The modal analysis is used to determine the vibration characteristics (natural frequencies and mode shapes) of the foundation. Block Lanczos Mode extraction method was carried out and 20 modes were extracted. The first three modal frequencies of the system are obtained in the range of 0.4755 to 0.54 Hz. The first three modal frequencies are within the required frequency range furnished in the specification. The element stress diagram and the deformed mode shapes obtained from the analysis are shown in Fig. 14.

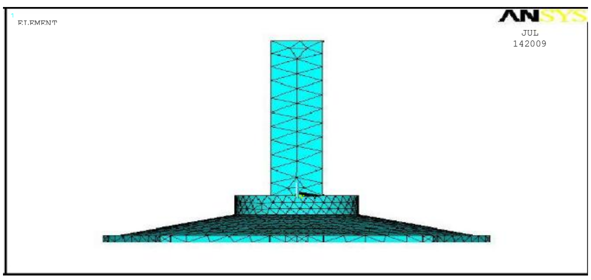


Fig. 14.Element stress contour and the deformed mode shape

2. Conclusions

Machine foundations transmit dynamic loads in addition to the static loads to the supporting soil. It's important to separate the natural frequency of the soil-foundation system from the operating frequency of the machine. In this paper, three distinct case studies were considered.

- In the first two case-studies there was a limitation on the foundation dimension; hence the soil conditions were altered to satisfy the required dynamic properties.

- In the final case, a wind-turbine foundation was designed to a given site characteristics. This paper explored the advantages of using Finite Element analysis in addition to classical tables and analytical solutions, to analysis and design foundations subjected to machine induced vibrations.

- These studies have shown the importance of validation of analytical solutions with FE simulations and the indispensable need for Finite Element analysis in dynamic analysis of machine foundations.

- However, an often overlooked step in machine foundation design is the post-construction observation of the foundation performance and its comparison with the predicted foundation behavior. Such comparisons are needed to calibrate new analysis procedures - an essential task in view of the simplifying assumptions on which even sophisticated formulations arebased.

It's important to focus on the post-construction observation and continuous monitoring to improve the validity of the analytical solutions and FE analysis in the design process and to incorporate and improve performance based design of machine foundations.

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